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SUBMANIFOLDS OF A RIEMANNIAN MANIFOLD ADMITTING A TYPE OF RICCI QUARTER-SYMMETRIC METRIC CONNECTION *

Abul Kalam Mondal

Abstract. The aim of the present paper is to study submanifolds of a Riemannian manifold admitting a type of Ricci quarter-symmetric metric connection. We have proved that the induced connection is also a Ricci quarter-symmetric metric connection. We have also considered the mean curvature and the shape operator of the submanifold with respect to the Ricci quarter-symmetric metric connection. We have obtained the Gauss, Codazzi and Ricci equations with respect to the Ricci quarter-symmetric metric connection. Finally, we have considered the totally geodesicness and obtained the relation between the sectional curvatures of the manifold and its submanifold with respect to the Ricci quarter-symmetric metric connection.

Keywords. Riemannian manifold; submanifolds; metric connection; curvature.

1. Introduction

Let ∇ be a linear connection in an n -dimensional differentiable manifold M . The torsion tensor T and the curvature tensor R of ∇ are given respectively by

$$T(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y]$$

and

$$R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X, Y]} Z.$$

The connection ∇ is symmetric if its torsion tensor T vanishes, otherwise it is called non-symmetric. The connection ∇ is a metric connection if there is a Riemannian metric g in M such that $\nabla g = 0$, otherwise it is called non-metric. It is well known that a linear connection is symmetric and metric if and only if it is the Levi-Civita

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connection. In 1975, S. Golab[8] introduced the notion of a quarter-symmetric linear connection in a differentiable manifold. In 1980, R. S. Mishra and S. N. Pandey [11] deduced some properties of the Riemannian, Kaehlerian and Sasakian manifolds that admits quarter-symmetric metric connection. In 1972, T. Imai[10] found some properties of a Riemannian manifold and hypersurfaces of a Riemannian manifold with a semisymmetric metric connection. In 1976, Z. Nakao[13] studied submanifolds of a Riemannian manifold with semisymmetric metric connection. Also in 1994, Agashe and Chafle[1] studied submanifolds of a Riemannian manifold with a semi-symmetric non-metric connection. In 2010, C. Ozgur[14], studied on submanifolds of a Riemannian manifold with a semi-symmetric non-metric connection. Also Zhao et al ([9],[16],[17]) studied on Riemannian manifolds with Quarter-symmetric metric connection. De et al ([7],[5],[6],[3],[12], [15]) studied Quarter-symmetric and Ricci Quarter-symmetric metric connections in Riemannian and Contact manifolds. Later in 2000, S. Ali and R. Nivas[2] studied on submanifolds immersed in a manifold with quarter-symmetric metric connection.

A linear connection is said to be a Ricci quarter-symmetric connection if its torsion tensor T is of the form

$$T(X, Y) = \pi(Y)QX - \pi(X)QY,$$

where π is a 1-form and Q is the Ricci tensor operator defined by

$$g(QX, Y) = S(X, Y),$$

where S is the Ricci tensor of type $(0, 2)$.

Motivated by these studies, in this paper we study submanifolds of a Riemannian manifold admitting a type of Ricci quarter-symmetric metric connection.

The present paper is organized as follows: After the preliminaries, in Section 3, we consider submanifold of a Riemannian manifold endowed with a Ricci quarter-symmetric metric connection and show that the induced connection on a submanifold of a Riemannian manifold with a Ricci quarter-symmetric metric connection is also a Ricci quarter-symmetric metric connection. We also show that the mean curvature vector of the Riemannian manifold with respect to the Levi-Civita connection and Ricci quarter-symmetric metric connection coincide if and only if the scalar curvature vanishes. In the last part of this section we prove that the shape operators with respect to the Levi-Civita connection are simultaneously diagonalizable if and only if the shape operators with respect to the Ricci quarter-symmetric metric connection are simultaneously diagonalizable. Finally, we study the Gauss and Codazzi equation with respect to the Ricci quarter-symmetric metric connection and prove that the normal connection ∇^\perp is flat if and only if all second fundamental tensors with respect to the Ricci quarter-symmetric and the Levi-Civita connection are simultaneously diagonalizable and also if the submanifold is totally geodesic with respect to the Ricci quarter-symmetric metric connection, then the sectional curvature of the manifold and its submanifold are identical.

2. Preliminaries

Let \bar{M} be an m -dimensional manifold with a Riemannian metric g and $\bar{\nabla}$ is the Levi-Civita connection on \bar{M} [18]. We define a linear connection ∇^* on \bar{M} by

$$(2.1) \quad \nabla_X^* Y = \bar{\nabla}_X Y + \pi(Y)QX - S(X, Y)\rho$$

for arbitrary vector fields X, Y of \bar{M} , where ρ is the vector field defined by $g(X, \rho) = \pi(X)$ and Q is the Ricci operator defined by $S(X, Y) = g(QX, Y)$, S is the Ricci tensor of $(0, 2)$ -type.

Using (2.1), the torsion tensor T^* with respect to the connection ∇^* is given by

$$(2.2) \quad T^*(X, Y) = \pi(Y)QX - \pi(X)QY.$$

A linear connection ∇^* satisfying the condition (2.2) is called a Ricci quarter-symmetric connection. Also using (2.1), we have

$$(\nabla_Z^* g)(X, Y) = (\bar{\nabla}_Z g)(X, Y) = 0.$$

Hence the connection is a metric connection.

We denote by R^* the curvature tensor of \bar{M} with respect to the Ricci quarter-symmetric metric connection ∇^* . So we have

$$(2.3) \quad \begin{aligned} R^*(X, Y)Z &= \nabla_X^* \nabla_Y^* Z - \nabla_Y^* \nabla_X^* Z - \nabla_{[X, Y]}^* Z \\ &= \bar{R}(X, Y)Z + (\bar{\nabla}_X \pi)(Z)QY - (\bar{\nabla}_Y \pi)(Z)QX \\ &\quad + \pi(Z)\{(\bar{\nabla}_X Q)Y - (\bar{\nabla}_Y Q)X\} + \{(\bar{\nabla}_Y S)(X, Z) \\ &\quad - (\bar{\nabla}_X S)(Y, Z)\}\rho + S(X, Z)\bar{\nabla}_Y \rho - S(Y, Z)\bar{\nabla}_X \rho \\ &\quad + \pi(Z)\{\pi(QY)QX - \pi(QX)QY + S(Y, QX)X \\ &\quad - S(X, QY)Y\} + \pi(\rho)\{S(X, Z)QY - S(Y, Z)QX\} \\ &\quad + \{S(Y, Z)S(X, \rho) - S(X, Z)S(Y, \rho)\}\rho, \end{aligned}$$

where $\bar{R}(X, Y)Z = \bar{\nabla}_X \bar{\nabla}_Y Z - \bar{\nabla}_Y \bar{\nabla}_X Z - \bar{\nabla}_{[X, Y]} Z$ is the curvature tensor of the manifold with respect to the Levi-Civita connection $\bar{\nabla}$. The Riemannian curvature tensors of the connections ∇^* and $\bar{\nabla}$ are defined by

$$R^*(X, Y, Z, W) = g(R^*(X, Y, Z), W)$$

and

$$\bar{R}(X, Y, Z, W) = g(\bar{R}(X, Y, Z), W)$$

respectively.

3. submanifolds of a Riemannian manifold with a Ricci quarter-symmetric metric connection

Let M be an n -dimensional submanifold of a Riemannian manifold \bar{M} with a Ricci quarter-symmetric metric connection. Decomposing the vector field ρ on M uniquely into their tangent and normal components ρ^T and ρ^\perp respectively we have

$$\rho = \rho^T + \rho^\perp.$$

The Gauss formula for a submanifold M of a Riemannian manifold \bar{M} with respect to the Riemannian connection $\bar{\nabla}$ is given by

$$(3.1) \quad \bar{\nabla}_X Y = \nabla_X Y + m(X, Y),$$

where X and Y are vector fields tangent to M and m is the second fundamental form of M in \bar{M} . If $m = 0$, then M is called totally geodesic with respect to the Riemannian connection. $H = \frac{1}{n} \text{trace } m$ is called the mean curvature vector of the submanifold. If $H = 0$, then M is called minimal. For the second fundamental form m , the covariant derivative of m is defined by

$$(\bar{\nabla}_X m)(Y, Z) = \nabla_X^\perp m(Y, Z) - m(\nabla_X Y, Z) - m(Y, \nabla_X Z).$$

for any vector field X tangent to M . $\bar{\nabla}$ is called the Van der Waerden-Bortolotti connection of M , that is, $\bar{\nabla}$ is the connection in $TM \oplus T^\perp M$ built with ∇ and ∇^\perp [4].

Let $\dot{\nabla}$ be the induced connection from the Ricci quarter-symmetric metric connection. We define

$$(3.2) \quad \nabla_X^* Y = \dot{\nabla}_X Y + \dot{m}(X, Y),$$

where \dot{m} is the induced second fundamental form.

The equation (3.2) is the Gauss equation with respect to the Ricci quarter-symmetric metric connection ∇^*

Using (2.1), from (3.1) and (3.2) we have

$$(3.3) \quad \begin{aligned} \dot{\nabla}_X Y + \dot{m}(X, Y) &= \nabla_X Y + m(X, Y) + \pi(Y)QX \\ &\quad - S(X, Y)\rho^T - S(X, Y)\rho^\perp. \end{aligned}$$

Now taking the tangential and normal parts we have

$$(3.4) \quad \dot{\nabla}_X Y = \nabla_X Y + \pi(Y)QX - S(X, Y)\rho^T$$

and

$$(3.5) \quad \dot{m}(X, Y) = m(X, Y) - S(X, Y)\rho^\perp.$$

If $\dot{m} = 0$, then M is called totally geodesic with respect to the Ricci quarter-symmetric connection.

From (3.4), we have

$$(3.6) \quad \dot{T}(X, Y) = \dot{\nabla}_X Y - \dot{\nabla}_Y X - [X, Y] = \pi(Y)QX - \pi(X)QY,$$

where \dot{T} is the torsion tensor of M with respect to $\dot{\nabla}$ and X, Y are vector fields tangent to M .

Moreover using (3.4) we have

$$(3.7) \quad (\dot{\nabla}_X g)(Y, Z) = (\nabla_X g)(Y, Z).$$

In view of (2.1), (3.4), (3.6) and (3.7) we can state the following:

Theorem 3.1. *The induced connection on a submanifold of a Riemannian manifold with a Ricci quarter-symmetric metric connection is also a Ricci quarter-symmetric metric connection.*

Let $\{e_1, e_2, \dots, e_n\}$ be an orthonormal basis of the tangent space of M . We define the mean curvature vector \dot{H} of M with respect to the Ricci quarter-symmetric metric connection $\dot{\nabla}$ by

$$\dot{H} = \frac{1}{n} \sum_{i=1}^n \dot{m}(e_i, e_i).$$

So from (3.5), we find

$$\dot{H} = H - \frac{r}{n} \rho^\perp.$$

If $H = 0$, then M is called minimal with respect to the Ricci quarter-symmetric metric connection.

So we have the following result:

Theorem 3.2. *If M be an n -dimensional submanifold of an m -dimensional Riemannian manifold \bar{M} , then the mean curvature vector of M with respect to the Levi-Civita connection and the Ricci quarter-symmetric metric connection coincide if and only if the scalar curvature vanishes.*

Let N be a normal vector field on M . From (2.1), we have

$$(3.8) \quad \nabla_X^* N = \bar{\nabla}_X N + \pi(N)QX.$$

The usual Weingarten formula is given by

$$(3.9) \quad \bar{\nabla}_X N = -A_N X + \nabla_X^\perp N, \quad N \in T^\perp(M)$$

where $-A_N X$ and $\nabla_X^\perp N$ are the tangential and normal parts of $\bar{\nabla}_X N$. From (3.8) and (3.9) we get

$$(3.10) \quad \nabla_X^* N = -\dot{A}_N X + \nabla_X^\perp N, \quad \text{where } \dot{A}_N = (A_N - \pi(N)Q)I,$$

which is the Weingarten formulae for a submanifold of a Riemannian manifold with respect to the Ricci quarter-symmetric metric connection.

Since A_N is symmetric, it is easy to see that

$$g(\dot{A}_N X, Y) = g(X, \dot{A}_N Y)$$

and

$$(3.11) \quad g([\dot{A}_N, \dot{A}_L]X, Y) = g(X, [A_N, A_L]Y),$$

where $[\dot{A}_N, \dot{A}_L] = \dot{A}_N \dot{A}_L - \dot{A}_L \dot{A}_N$, $[A_N, A_L] = A_N A_L - A_L A_N$, N and L are unit normal vector fields on M .

Theorem 3.3. *If M be an n -dimensional submanifold of an m -dimensional Riemannian manifold \bar{M} admitting the Ricci quarter-symmetric metric connection, then the shape operators with respect to Levi-Civita connection are simultaneously diagonalizable if and only if the shape operators with respect to the Ricci quarter-symmetric metric connection are simultaneously diagonalizable.*

4. Gauss and codazzi equation with respect to the Ricci quarter-symmetric metric connection

We denote the curvature tensor of \bar{M} with respect to the Ricci quarter-symmetric metric connection ∇^* by

$$R^*(X, Y)Z = \nabla_X^* \nabla_Y^* Z - \nabla_Y^* \nabla_X^* Z - \nabla_{[X, Y]}^* Z$$

and that of M with respect to the induced Ricci quarter-symmetric metric connection $\dot{\nabla}$ by

$$\dot{R}(X, Y)Z = \dot{\nabla}_X \dot{\nabla}_Y Z - \dot{\nabla}_Y \dot{\nabla}_X Z - \dot{\nabla}_{[X, Y]} Z.$$

We shall now find the equation of Gauss-Codazzi with respect to the Ricci quarter-symmetric metric connection.

$$\begin{aligned} R^*(X, Y)Z &= \nabla_X^* \nabla_Y^* Z - \nabla_Y^* \nabla_X^* Z - \nabla_{[X, Y]}^* Z \\ &= \dot{\nabla}_X \dot{\nabla}_Y Z + \dot{m}(X, \dot{\nabla}_Y Z) - \dot{A}_{\dot{m}(Y, Z)} X + \nabla_X^\perp \dot{m}(Y, Z) \\ &\quad - \dot{\nabla}_Y \dot{\nabla}_X Z - \dot{m}(Y, \dot{\nabla}_X Z) + \dot{A}_{\dot{m}(X, Z)} Y - \nabla_Y^\perp \dot{m}(X, Z) \\ &\quad - \dot{\nabla}_{[X, Y]} Z - \dot{m}([X, Y], Z) \\ &= \dot{R}(X, Y)Z - \dot{A}_{\dot{m}(Y, Z)} X + \dot{A}_{\dot{m}(X, Z)} Y \\ &\quad + \dot{m}(X, \dot{\nabla}_Y Z) - \dot{m}(Y, \dot{\nabla}_X Z) - \dot{m}([X, Y], Z) \\ &\quad + \nabla_X^\perp \dot{m}(Y, Z) - \nabla_Y^\perp \dot{m}(X, Z). \end{aligned} \quad (4.1)$$

Taking account of (3.10), we have

$$\begin{aligned}
 R^*(X, Y)Z &= \dot{R}(X, Y)Z - A_{\dot{m}(Y, Z)}X - \pi(\dot{m}(Y, Z))X + A_{\dot{m}(X, Z)}Y \\
 &\quad + \pi(\dot{m}(X, Z))Y + \dot{m}(X, \dot{\nabla}_Y Z) - \dot{m}(Y, \dot{\nabla}_X Z) \\
 (4.2) \quad &\quad - \dot{m}([X, Y], Z) + \nabla_X^\perp \dot{m}(Y, Z) - \nabla_Y^\perp \dot{m}(X, Z).
 \end{aligned}$$

Since $g(A_N X, Y) = g(h(X, Y), N)$, using (3.5) we obtain

$$\begin{aligned}
 R^*(X, Y, Z, W) &= \dot{R}(X, Y, Z, W) - g(A_{\dot{m}(Y, Z)}X, W) + g(A_{\dot{m}(X, Z)}Y, W) \\
 &\quad + \pi(\dot{m}(Y, Z))g(QX, W) - \pi(\dot{m}(X, Z))g(QY, W) \\
 &= \dot{R}(X, Y, Z, W) - g(m(Y, Z), m(X, W)) \\
 &\quad + g(m(X, Z), m(Y, W)) + S(Y, Z)\pi(m(X, W)) \\
 &\quad - S(X, Z)\pi(m(Y, W)) + S(X, W)\pi(m(Y, Z)) \\
 &\quad - S(Y, W)\pi(m(X, Z)) + \pi(\rho^\perp)[S(X, Z)S(Y, W) \\
 (4.3) \quad &\quad - S(Y, Z)S(X, W)],
 \end{aligned}$$

where W is a tangent vector field on M .

From (4.2), the normal component of $R^*(X, Y)Z$ is given by

$$\begin{aligned}
 (R^*(X, Y)Z)^\perp &= \dot{m}(X, \dot{\nabla}_Y Z) - \dot{m}(Y, \dot{\nabla}_X Z) - \dot{m}([X, Y], Z) \\
 &\quad + \nabla_X^\perp \dot{m}(Y, Z) - \nabla_Y^\perp \dot{m}(X, Z) \\
 &= (\nabla_X^\perp \dot{m})(Y, Z) - (\nabla_Y^\perp \dot{m})(X, Z) \\
 (4.4) \quad &\quad + \pi(Y)\dot{m}(QX, Z) - \pi(X)\dot{m}(QY, Z),
 \end{aligned}$$

where $(\nabla_X^\perp \dot{m})(Y, Z) = \nabla_X^\perp \dot{m}(Y, Z) - \dot{m}(\dot{\nabla}_X Y, Z) - \dot{m}(\dot{\nabla}_Y X, Z)$.

It is called the van der Waerden-Bortolotti connection with respect to the Ricci quarter-symmetric metric connection.

Also the equation (4.4) is the equation of Codazzi with respect to the Ricci quarter-symmetric metric connection.

From (3.2) and (3.10), we get

$$\begin{aligned}
 \nabla_X^* \nabla_Y^* N_1 &= -\dot{A}_{\nabla_Y^\perp N_1} X - \dot{m}(\dot{A}_{N_1} Y, X) \\
 (4.5) \quad &\quad + \nabla_X^\perp \nabla_Y^\perp N_1 - \dot{\nabla}_X \dot{A}_{N_1} Y,
 \end{aligned}$$

$$\begin{aligned}
 \nabla_Y^* \nabla_X^* N_1 &= -\dot{A}_{\nabla_X^\perp N_1} Y - \dot{m}(\dot{A}_{N_1} X, Y) \\
 (4.6) \quad &\quad + \nabla_Y^\perp \nabla_X^\perp N_1 - \dot{\nabla}_Y \dot{A}_{N_1} X
 \end{aligned}$$

and

$$(4.7) \quad \nabla_{[X, Y]}^* N_1 = -\dot{A}_{N_1}[X, Y] + \nabla_{[X, Y]}^\perp N_1.$$

So using (4.5)-(4.7), we have

$$(4.8) \quad \begin{aligned} R^*(X, Y, N_1, N_2) &= R^\perp(X, Y, N_1, N_2) - g(\dot{m}(\dot{A}_{N_1}Y, X), N_2) \\ &\quad + g(\dot{m}(\dot{A}_{N_1}X, Y), N_2), \end{aligned}$$

where N_1 and N_2 are normal vector fields on M . Hence in view of (3.5) and (3.10) the equation (4.8) turns into

$$(4.9) \quad \begin{aligned} R^*(X, Y, N_1, N_2) &= R^\perp(X, Y, N_1, N_2) - g(m(A_{N_1}Y, X), N_2) \\ &\quad + S(A_{N_1}Y, X)g(\rho^\perp, N_2) + g(m(A_{N_1}X, Y), N_2) \\ &\quad - S(A_{N_1}X, Y)g(\rho^\perp, N_2) \\ &= R^\perp(X, Y, N_1, N_2) + g([N_1, N_2]X, Y). \end{aligned}$$

The equation (4.9) is the equation of Ricci with respect to the Ricci quarter-symmetric metric connection.

If \bar{M} is a space of constant curvature c with respect to the connection $\bar{\nabla}$, then the equation (2.2) reduce to

$$(4.10) \quad \begin{aligned} R^*(X, Y)Z &= c\{g(y, Z)X - g(X, Z)Y\} + (\bar{\nabla}_X\pi)(Z)QY - (\bar{\nabla}_Y\pi)(Z)QX \\ &\quad + \pi(Z)\{(\bar{\nabla}_XQ)Y - (\bar{\nabla}_YQ)X\} + \{(\bar{\nabla}_Y S)(X, Z) \\ &\quad - (\bar{\nabla}_X S)(Y, Z)\}\rho + S(X, Z)\bar{\nabla}_Y\rho - S(Y, Z)\bar{\nabla}_X\rho \\ &\quad + \pi(Z)\{\pi(QY)QX - \pi(QX)QY + S(Y, QX)X \\ &\quad - S(X, QY)\} + \pi\{S(X, Z)QY - S(Y, Z)QX\} \\ &\quad + \{S(Y, Z)S(X, \rho) - S(X, Z)S(Y, \rho)\}\rho. \end{aligned}$$

From (4.10) we have $R^*(X, Y, N_1, N_2) = 0$. Therefore using (3.11) and (4.9) we obtain

$$R^\perp(X, Y, N_1, N_2) = g([N_2, N_1]X, Y) = g([N_2^*, N_1^*]X, Y).$$

Hence we can state the following theorem:

Theorem 4.1. *If M be an n -dimensional submanifold of an m -dimensional space of constant curvature $\bar{M}(c)$ admitting Ricci quarter-symmetric metric connection, then the normal connection ∇^\perp is flat if and only if all second fundamental tensors with respect to the Ricci quarter-symmetric and the Levi-Civita connection are simultaneously diagonalizable.*

From the equation (4.3) we have

$$R^*(X, Y, Y, X) = \dot{R}(X, Y, Y, X) - g(m(X, X), m(Y, Y))$$

$$\begin{aligned}
& +g(m(X, Y), m(Y, X)) + S(Y, Y)\pi(m(X, X)) \\
& -S(X, X)\pi(m(Y, Y)) - S(X, Y)\pi(m(X, Y)) \\
& -S(Y, X)\pi(m(X, Y)) + \pi(\rho^\perp)[S(X, Y)S(Y, X) \\
& -S(Y, Y)S(X, X)].
\end{aligned}
\tag{4.11}$$

Now if the sectional curvature of \bar{M} and M at a point $p \in \bar{M}$ with respect to the Ricci quarter-symmetric metric connection is denoted by κ^* and $\dot{\kappa}$ respectively, then the equation (4.11) reduce to

$$\begin{aligned}
\kappa^* &= \dot{\kappa} - g(m(X, X), m(Y, Y)) + g(m(X, Y), m(Y, X)) \\
& +S(Y, Y)\pi(m(X, X)) - S(X, X)\pi(m(Y, Y)) \\
& -S(X, Y)\pi(m(X, Y)) - S(Y, X)\pi(m(X, Y)) \\
& +\pi(\rho^\perp)[S(X, Y)S(Y, X) - S(Y, Y)S(X, X)].
\end{aligned}
\tag{4.12}$$

If we consider M is totally geodesic with respect to the Ricci quarter-symmetric metric connection, then from (3.5) we get $m(X, Y) = S(X, Y)\rho^\perp$ and using this result, the equation (4.12) becomes

$$\kappa^* = \dot{\kappa}.$$

Hence we have the following theorem:

Theorem 4.2. *Let M be an n -dimensional submanifold of an m -dimensional Riemannian manifold \bar{M} admitting a Ricci quarter-symmetric metric connection. If M is totally geodesic with respect to the Ricci quarter-symmetric metric connection, then the sectional curvatures κ^* and $\dot{\kappa}$ of \bar{M} and M (resp.) are identical.*

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Abul Kalam Mondal
 Department of Mathematics
 Acharya Prafulla Chandra College,
 New Barrackpore, Kolkata-700131
 West Bengal, India
 kalam_ju@yahoo.co.in